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ORIGINAL CONTRIBUTION

An Application of Modified Path Matrix Approach for Detection of Isomorphism Among Epicyclic Gear Trains

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Abstract The identification of isomorphism in epicyclic gear trains has been found a lot of attention by researchers for the last few years. Various methods have been suggested by different authors for the detection of isomorphism in planer kinematic chains and epicyclic gear trains (EGTs), but everyone has found some difficulties to address new issues. In this paper, a modified path matrix approach was presented in order to compare all the distinct geared kinematic mechanisms. A new method based on the matrix approach and corresponding train values is required to identify isomorphism among epicyclic gear trains and their mechanisms. The proposed method was examined on the basis of various examples from four-link, five-link, six-link, and eight-link one-degree-of-freedom EGTs and six-link two-degree-of-freedom EGTs. All the examples have been found satisfactory results with existing literature.

Keywords Epicyclic gear trains · Modified path matrix · Isomorphism · Pair value · Train value

Introduction

Isomorphism identification in epicyclic gear trains (EGTs) has found a lot of attention for doing analysis and synthesis of mechanism during the last few decades. It is one of the basic ways to improve the performance of the mechanism by graph theory. Graph theory is being used for analyzing the graphs conventionally. With the help of graph theory, the kinematic diagram is converted into graphical form and then, by using various methods of identifying the isomorphism among geared kinematic mechanisms.

Topological synthesis and analysis of planer and geared kinematic mechanisms is very complex process. The comprehensive study of structural modeling of epicyclic gear trains was reported by various authors as presented here.

Various mathematical and graphical methods for structural analysis, force, and torque characteristic of epicyclic gear trains were initially presented by Levai [1]. Buchsbaum and Freudenstein [2] developed a method for classification and enumeration of mechanism according to kinematic structures using network theorems and combinatorial analysis. A novel method for kinematic structure of epicyclic gear drives was presented by application of Boolean algebra [3]. A procedure based on the functional constraints for analysis of the graphs was computed with one-DOF epicyclic gear trains [4]. Another efficient method for the topological synthesis of fractionated geared kinematic mechanisms was developed by Chen and Yao [5]. An efficient and simple algorithm was presented for the conceptual design of geared robot manipulators for proper mechanical structures [6]. Tsai [7, 8] reported new method based on random number technique for determination of isomorphic values in EGTs with different degrees of freedom. Hsu et al. [9] had developed an efficient model for kinematic study of the epicyclic spur-gear trains with different links by using graph applications. Also, it has been suggested a new idea for the generation of graphs for kinematic structure of EGTs [10]. An algorithm based on graph theory was suggested about the identification of embedded structure for structural synthesis in planetary gear trains [11]. A new method based on edge permutation

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concept which was used to identify the non-isomorphic graphs in epicyclic gear trains [12]. Another method based on kinematic fractionation was developed by researchers for the kinematic propagation path for various links in geared mechanisms as given in detail in [13–15]. The new approach was developed by application of loop technique and Hamming number technique to check the rotational and displacement isomorphism in planetary gear trains [16]. Another technique of genetic algorithm approach was suggested for detection of isomorphism among EGTs and their mechanisms [17]. Shin and Krishnamurty [18] introduced an efficient procedure based on standard code technique for generation of epicyclic gear trains.

Rao et al. [19] reported on the synthesis of kinematic chains and their mechanism. Bedi and Sanyal [20] developed a new technique for detect distinct mechanism of kinematic chains on the basis of link-joint matrix. Hasan et al. [21] also suggested about the concept of joint–joint matrix method for identification of distinct mechanisms of kinematic chains. Medapati et al. [22] discussed a new algorithm for generation of KCs and their mechanism up to 11 links with a higher degree of freedom. Hamming number technique was again reported by Saini and Singh [23] for detection of isomorphism in nine-link KCs up to two degrees of freedom.

Xue et al. [24] reported a review of graph theory applications in structural synthesis of planetary gear trains. El-gayyar et al. [25] developed acyclic graph model for the structural synthesis and analysis of EGTs up to 12 links. Esmail [26, 27] presented a new approach based on the basis of the graph theory for kinematic analysis of epicyclic gear mechanism (EGM) by using the kinematic nomographs. It was more efficient for enumeration of feasible clutching sequences of epicyclic-type transmission mechanisms. A novel automatic design model of epicyclic gear mechanism based on graph theory and ant algorithm was reported by Ping and Pei [28]. Ravisankar and Mruthyunjaya [29] suggested graph-based fully computerized method for the structural synthesis of one-DOF-geared kinematic chains up to four geared pair. Tsai et al. [30] suggested about control techniques to kinematic analysis of planetary gear systems by block diagrams. Yang et al. introduced a mixed model based on ant algorithm and mapping property for identification of isomorphism in epicyclic gear mechanisms [31]. A novel algorithm on the concept of graph theory was developed for structural synthesis and analysis of planetary gear trains [32]. Rao and Rao [33] suggested about isomorphism identification and compactness in EGTs by application of Hamming matrices and Moment matrices. Efficiency study of epicyclic gear trains was also conducted for critical design, optimization and operation of the system on the concept of virtual power [34] and various methods for efficiency and kinematics of EGTs with two DOFs discussed in detail by researchers in [35, 36].

Lohumi et al. [37] proposed a method based on path matrix algorithm to detect isomorphism in planer kinematic chains. A general expression for visualization of velocities, torques, and power flow in the epicycic gear train was suggested on the basis of nomographs as given in details [38]. A new algorithm has been introduced for automatic computation study of epicyclic gear trains by Eashy and Gayyar [39]. The automatic evaluation of displacement graphs was presented by a new approach 'graphical code method'. Analysis of the efficiency and transmission ratio with various ranges of four-, five-, and six-link planetary gear trains was studied in detail [40]. The graphs of these trains to determine all the structurally distinct kinematic inversions, then obtained all the constructive solutions resulting from every possible combination of gear type and configuration. Rajasri et al. [41] suggested about identification of isomorphism in planetary gear trains based on adjacency symmetrical matrix. A Hamming number technique was used to identify the isomorphism in PGTs. Peng et al. [42] proposed a topological synthesis of planetary gear trains (PGTs) based on the variable structure with multiple operating degrees of freedom. Pennestri et al. [43] dealt about graph-based algorithm for study of mechanisms and gear trains. An automatic approach for structural synthesis of EGTs came to main focus by Ding et al. [44]. It was required to canonical adjacency matrices and characteristics number strings to identify isomorphism for synthesis and analysis of geared mechanisms. The hamming number technique is one of the important steps to detect the isomorphism in geared kinematic mechanisms. The new formation of one-degree-of-freedom EGTs with up to seven links was presented by Rajasri et al. [45]. The descriptions of multicolor graph representation and structural synthesis of planar kinematic chains and geared kinematic chains based on single-open-chains were studies in details in [46]. Yang and Ding proposed a new algorithm for representation the graph models in planetary gear trains in [47]. Gao and Hu have introduced about kinematic analysis of planetary gear trains on the concept of graph theory [48].

Kamesh et al. [49] proposed a new methodology for detection of isomorphism in the kinematic chains and epicyclic gear trains on the basis of graph theory. It has given a new concept related to 'Net distance' for quantitative analysis of isomorphism in mechanisms. Another novel and easy method was proposed using a parameter like vertex incidence polynomial (VIP) to synthesize epicyclic gear mechanisms up to six links for detection of isomorphism. Identification of isomorphic graphs was developed by calculating VIP. Secondly, another parameter 'rotation index' (RI) was proposed to detect rotational

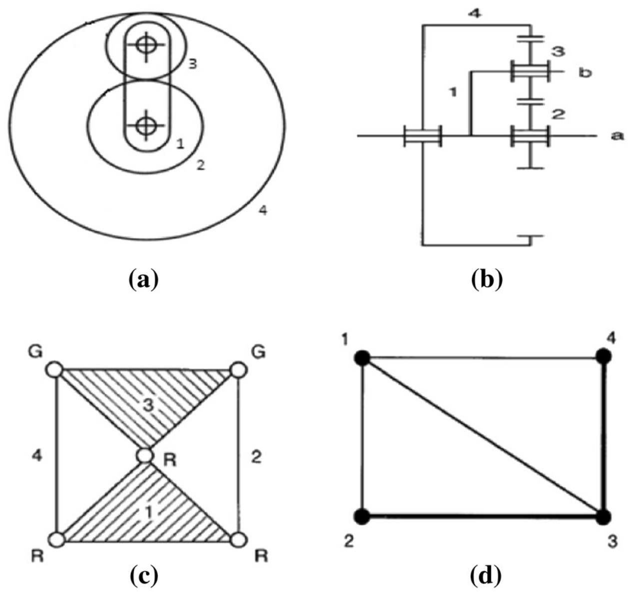


Fig. 1 **a** Kinematic diagram. **b** Functional schematic representation. **c** Kinematic schematic representation. **d** Graph representation of EGTs

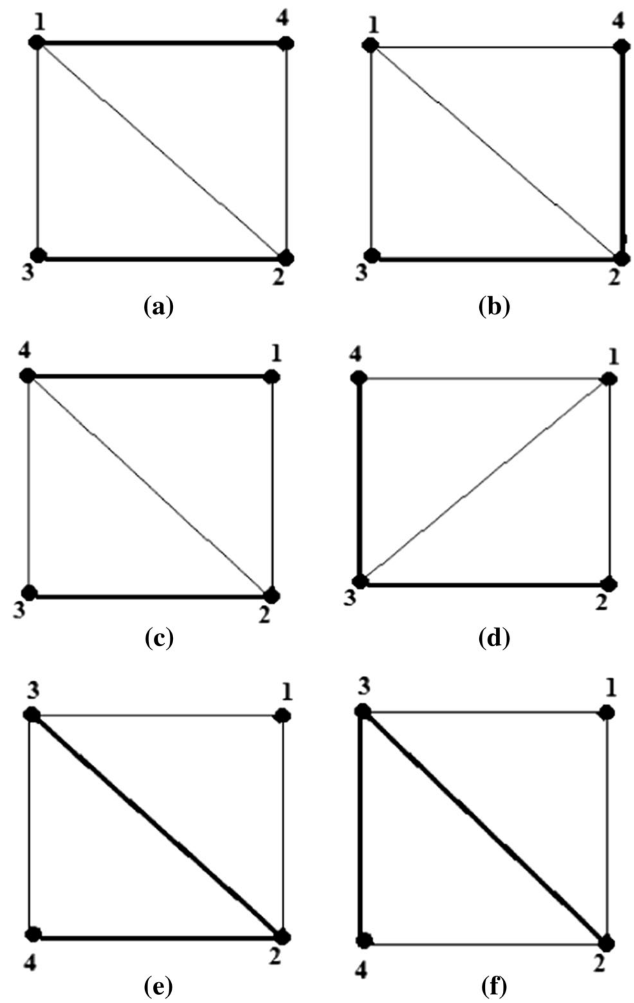


Fig. 2 Generation of different graphs of four-link EGTs with one DOF

isomorphism [50]. A new algorithm on the basis of fundamental circuit's rotation graphs has been proposed for detecting degenerate structure in PGTs [51].

Esmail [52] developed two methods are for kinematic study of planetary gear trains, viz. formula and graphical methods. The tree-graph approach was developed for detection of isomorphism in epicyclic gear trains and their mechanisms. The adjacency matrices of EGTs were calculated for identification of equivalent structures and synthesis analysis [53]. The analysis of kinematic chains with zero variety for 1–2 DOF epicyclic gear trains was presented by Souza et al. in detail in [54]. The concept and determination of mobility were given by introducing the definition of variety, minimal sets, and advantage of selecting a variety zero of epicyclic gear trains. The kinematic model of a planetary screw roller mechanism was suggested for fundamental analysis of rigid body kinematics, i.e., velocity of nut and transmission error of mechanism [55]. The brief discussion of graphs theory application and isomorphism detection in epicyclic gear trains was presented in [56, 57].

Finally, it has been observed from various critical research articles on the detection of isomorphism, kinematic analysis and several efficient methodologies for topological synthesis of epicyclic gear trains have taken main focus of study by researchers. The major difficult phase was found during modeling and synthesis of geared mechanisms that to identify the isomorphism in different generating gap of research.

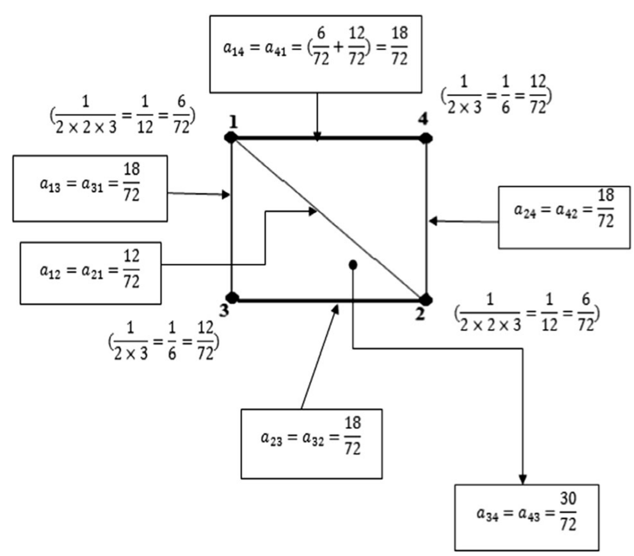


Fig. 3 Calculation of pair values and train value for graph-2(a)

Table 1 Least path value (LPV) descriptions between link 1 and 2 for EGTs-2(a)

Path no.	Descriptions of path	Value of path
1	1–2	(6/72) + (6/72) = (12/72)
2	1–3–2	(6/72) + (12/72) + (6/72) = (24/72)
3	1–4–2	(6/72) + (12/72) + (6/72) = (24/72)

Table 2 Pair values and train values for different graph of four-link one-DOF epicyclic gear trains

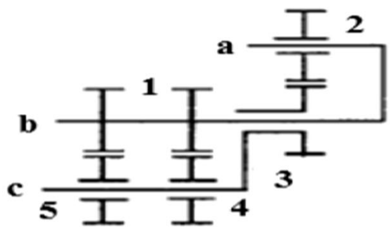
Fig. no.	Modified path matrix (MPM)	Pair value (PV)	Train value (TV)
2a	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 12 & 18 & 18 \\ 12 & 0 & 18 & 18 \\ 18 & 18 & 0 & 30 \\ 18 & 18 & 30 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 48 \\ 48 \\ 66 \\ 66 \end{bmatrix}$	$\left(\frac{228}{72}\right) = 3.167$
2b	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 13 & 21 & 21 \\ 13 & 0 & 16 & 16 \\ 21 & 16 & 0 & 30 \\ 21 & 16 & 30 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 55 \\ 45 \\ 57 \\ 57 \end{bmatrix}$	$\left(\frac{214}{72}\right) = 2.972$
2c	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 18 & 30 & 18 \\ 18 & 0 & 18 & 12 \\ 30 & 18 & 0 & 18 \\ 18 & 12 & 18 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 66 \\ 48 \\ 66 \\ 48 \end{bmatrix}$	$\left(\frac{228}{72}\right) = 3.167$
2d	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 21 & 13 & 21 \\ 21 & 0 & 16 & 30 \\ 13 & 16 & 0 & 16 \\ 21 & 30 & 16 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 55 \\ 57 \\ 45 \\ 57 \end{bmatrix}$	$\left(\frac{214}{72}\right) = 2.972$
2e	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 22 & 24 & 34 \\ 22 & 0 & 10 & 16 \\ 24 & 10 & 0 & 18 \\ 34 & 16 & 18 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 80 \\ 48 \\ 52 \\ 68 \end{bmatrix}$	$\left(\frac{248}{72}\right) = 3.444$
2f	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 24 & 22 & 34 \\ 24 & 0 & 10 & 18 \\ 22 & 10 & 0 & 16 \\ 34 & 18 & 16 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 80 \\ 52 \\ 48 \\ 68 \end{bmatrix}$	$\left(\frac{248}{72}\right) = 3.444$

Graphical Representation of Epicyclic Gear Trains

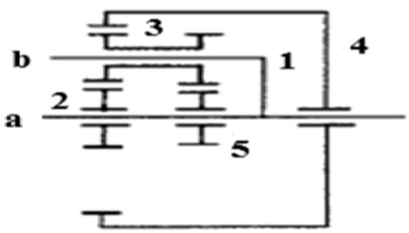
Structural synthesis and analysis of EGTs involve the development of a group of graphical representation of geared mechanisms for detection of isomorphism. Exactly identical graphs discarded with the detection for isomorphism. In this paper, isomorphism can be classified on the path matrix approach, number of links, type of kinematic pair, joint value, pair value and train value. The basic test criteria for identification of isomorphism are on the basis of similar numeric quantity of pair value and train values between two EGTs.

The major application of detection of isomorphism in epicyclic gear trains (EGTs) as well as kinematic chains (KCs) is to develop every possible mechanism derived from a given EGTs and KCs so that researcher has the liberty to select the best or optimum mechanism depending upon its application. A number of distinct epicyclic gear trains exist with a specified number of links and degree-of-freedom.

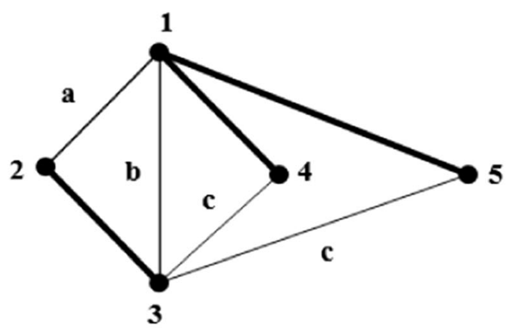
The topological graph of epicyclic gear trains can be achieved by using two lines as *continuous thin lines* (“—”)



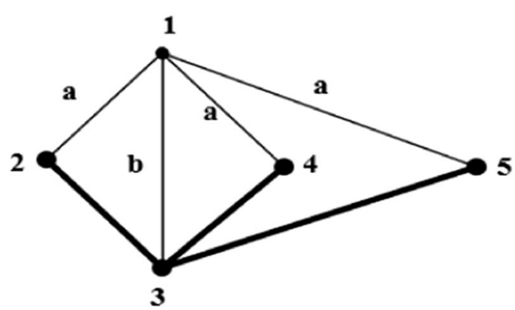
(a) 5-links EGT-1



(b) 5-links EGT-2



(c) Conventional graph of 5-link EGT-1

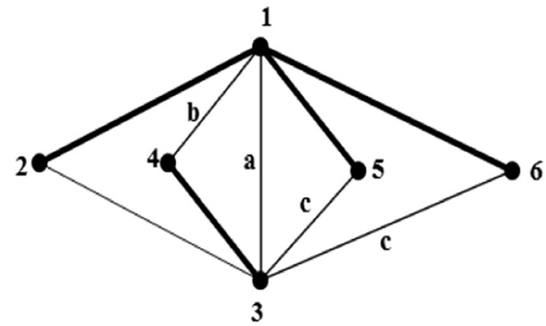


(d) Conventional graph of 5-links EGT-2

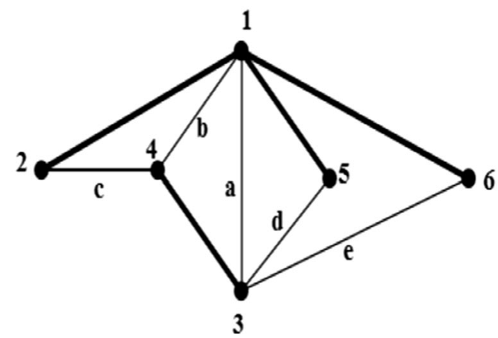
Fig. 4 **a** Five-link EGT-1. **b** Five-link EGT-2. **c** Conventional graph of five-link EGT-1. **d** Conventional graph of five-link EGT-2

to denote lower pairs (i.e., turning pair or prismatic pair) and *continuous thick lines* ("—")to denote higher pairs (i.e., gear pair) as shown in Fig. 1a–d.

So, for graphical study, an EGT can be converted into graphs and graphs are converted to an adjacency matrix or connectivity matrix or modified path matrix. The graphical representations of four-link EGTs are as given in Fig. 2. These four-link EGTs are generated by adding a vertex to the existing three-link EGT. There are $n(n-1)$ ways to



(a) Conventional Graph of 6-links EGT-1



(b) Conventional Graph of 6-links EGT-2

Fig. 5 **a** Conventional graph of six-link EGT-1. **b** Conventional graph of six-link EGT-2

add an additional vertex to the parent graph. So, we have got 6 graphs by adding a vertex in three-link EGTs [17]. It was also explained in different research papers [31, 45].

Definitions of Terminology

The various terminologies used for identification of the isomorphism in geared mechanisms have been explained as given below:

Degree of Kinematic Pair (DK)

It is numerical quantity to represent the connectivity of kinematic pair in geared kinematic mechanisms. For examples.

Degree of kinematic pair for lower pair (i.e., turning pair or prismatic pair) = 2, and degree of kinematic pair for higher pair (i.e., gear pair) = 3.

Joint Value (JV)

It is numerical quantity and the inverse of multiplication of degree of kinematic pair.

Table 3 Pair values and train values for different graph of five-link epicyclic gear trains

Fig. no.	Modified path matrix (MPM)	Pair value (PV)	Train value (TV)
4c	$\left(\frac{1}{72}\right)\begin{bmatrix} 0 & 14 & 5 & 14 & 14 \\ 14 & 0 & 15 & 26 & 26 \\ 5 & 15 & 0 & 15 & 15 \\ 14 & 26 & 15 & 0 & 26 \\ 14 & 26 & 15 & 26 & 0 \end{bmatrix}$	$\left(\frac{1}{72}\right)\begin{bmatrix} 47 \\ 81 \\ 50 \\ 81 \\ 81 \end{bmatrix}$	$\left(\frac{340}{72}\right) = 4.722$
4d	$\left(\frac{1}{432}\right)\begin{bmatrix} 0 & 99 & 35 & 99 & 99 \\ 99 & 0 & 80 & 152 & 152 \\ 35 & 80 & 0 & 80 & 80 \\ 99 & 152 & 80 & 0 & 152 \\ 99 & 152 & 80 & 152 & 0 \end{bmatrix}$	$\left(\frac{1}{432}\right)\begin{bmatrix} 432 \\ 483 \\ 275 \\ 483 \\ 483 \end{bmatrix}$	$\left(\frac{2156}{432}\right) = 4.991$

Table 4 Pair values and train values for different graph of five-link epicyclic gear trains

Fig. no.	Modified path matrix [MPM]	Pair value (PV)	Train value (TV)
5a	$\left(\frac{1}{432}\right)\begin{bmatrix} 0 & 76 & 13 & 76 & 76 & 76 \\ 76 & 0 & 81 & 148 & 148 & 148 \\ 13 & 81 & 0 & 81 & 81 & 81 \\ 76 & 148 & 81 & 0 & 148 & 148 \\ 76 & 148 & 81 & 148 & 0 & 148 \\ 76 & 148 & 81 & 148 & 148 & 0 \end{bmatrix}$	$\left(\frac{1}{432}\right)\begin{bmatrix} 317 \\ 601 \\ 337 \\ 601 \\ 601' \\ 601 \end{bmatrix}$	$\left(\frac{3058}{432}\right) = 7.0787$
5b	$\left(\frac{1}{432}\right)\begin{bmatrix} 0 & 76 & 22 & 40 & 76 & 76 \\ 76 & 0 & 94 & 108 & 148 & 148 \\ 22 & 94 & 0 & 54 & 90 & 90 \\ 40 & 108 & 54 & 0 & 112 & 112 \\ 76 & 148 & 90 & 112 & 0 & 148 \\ 76 & 148 & 90 & 112 & 148 & 0 \end{bmatrix}$	$\left(\frac{1}{432}\right)\begin{bmatrix} 290 \\ 574 \\ 350 \\ 426 \\ 574 \\ 574 \end{bmatrix}$	$\left(\frac{2788}{432}\right) = 6.4537$

Pair Value (PV)

It is the addition of all elements of a particular row or column of a given matrix.

Train Value (TV)

It is the addition of all the pair values of EGTs or it is addition of all row and column values of the matrix.

Least Path Value (LPV)

It is defined as the lowest sum of all joint value between a particular two kinematic links.

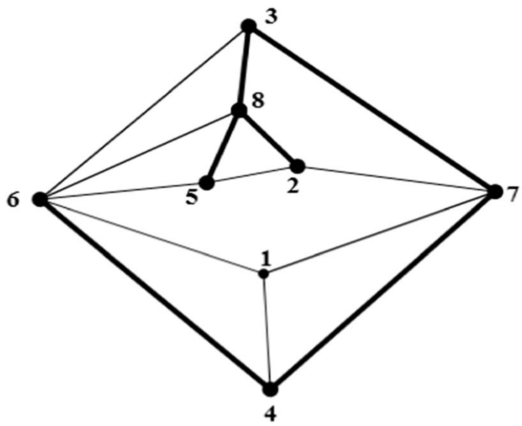
Modified Path Matrix (MPM)

The modified path matrix of EGTs is achieved by calculation of connectivity matrix which is a square and symmetrical matrix. Each element of modified path matrix '$[a_{ij}]$' is defined as the lowest sum of all joint value between a particular two link i and j. It is equal to zero if i is equal to j (i.e., $i = j$, then $a_{ij} = 0$) as given in Eq. (1) and modified path matrix as focused in Eq. (2).

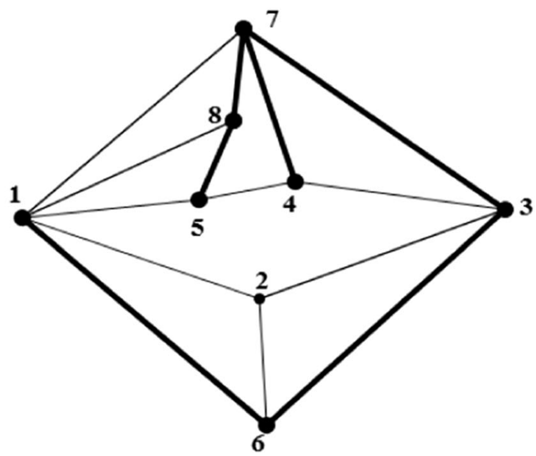
$$[a_{ij}] \begin{cases} \boldsymbol{Min\sum_{i}^{j=k} \frac{1}{\prod(L_k \;\; H_k)}} \textit{ i.e., Minimum summation of joint values:} \\ \textit{If vertex i connected to vertex j by lower-pair or higher-pair.} \\ \textit{Where, } L_k - \textit{Nodal value of lower pair (or turning} - \textit{pair),} \\ \quad H_k - \textit{Nodal value of higher pair(or geared} - \textit{pair).} \\ \\ \textit{0 otherwise (including i=j)} \end{cases} \quad (1)$$

$$[\mathrm{MPM}] = \begin{bmatrix} 0 & a_{12} & a_{13} & \ldots & a_{1n} \\ a_{21} & 0 & a_{23} & \ldots & a_{2n} \\ \ldots & \ldots & 0 & \ldots & \ldots \\ \ldots & \ldots & \ldots & 0 & \ldots \\ a_{n1} & a_{n2} & a_{n2} & \ldots & 0 \end{bmatrix} \quad (2)$$

Algorithm for Identification of Isomorphism



(a) Conventional Graph of 8-links EGT-1



(b) Conventional Graph of 8-links EGT-2

Fig. 6 **a** Conventional graph of eight-link EGT-1. **b** Conventional graph of eight-link EGT-2

The steps in the algorithm for identification of isomorphism in epicyclic gear trains are:

a. For each link or vertex in a graph, calculate 'degree of kinematic pair (DK)' from all other links or vertices.
b. Inverse multiplications of all degrees of kinematic pair will results 'joint value (JV)' for the link.
c. Compute the least path value (LPV) by application of the least joint value of the link.
d. Summation of all the elements of a particular row or column of a matrix will result 'pair value (PV)' of the epicyclic gear trains.
e. Sum of all the pair values of EGTs will give 'train value (TV)'.
f. 'Train value' of an EGT is used as a quantitative measure to compare with any other epicyclic gear trains.
g. If any two EGTs have the same 'train value', they are said to be 'isomorphic' else 'distinct'.

Illustrative Examples

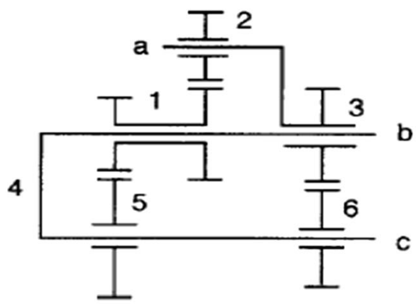
The proposed algorithm is examined on epicyclic gear trains with different linkage and degrees of freedom. The illustrative examples are as follows:

Example 1 Consider six-graph four-link one-DOF epicyclic gear trains shown in Fig. 2a–f.

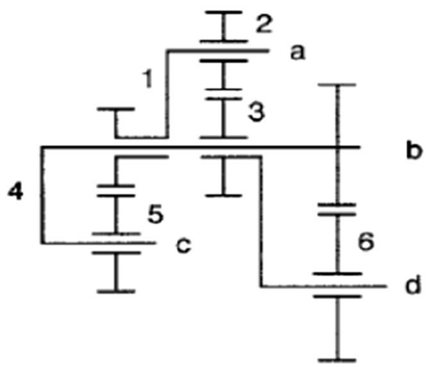
In the proposed algorithm, the elemental value of the modified path matrix for epicyclic gear trains is determined by the least path value (LPV). Summation of all the elements of a particular row or column of a matrix results

Table 5 Pair values and train values for different graph of eight-link epicyclic gear trains

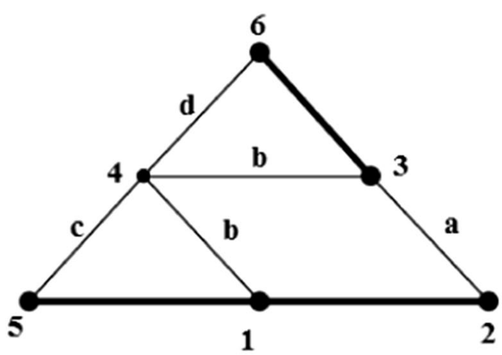
Fig. no.	Modified path matrix (MPM)	Pair value (PV)	Train value (TV)
6a	$\left(\frac{1}{432}\right)\begin{bmatrix} 0 & 102 & 87 & 78 & 99 & 63 & 66 & 71 \\ 102 & 0 & 68 & 72 & 72 & 81 & 48 & 44 \\ 87 & 68 & 0 & 57 & 68 & 33 & 36 & 32 \\ 78 & 72 & 57 & 0 & 69 & 33 & 36 & 41 \\ 99 & 72 & 68 & 69 & 0 & 45 & 81 & 44 \\ 63 & 81 & 33 & 33 & 45 & 0 & 45 & 17 \\ 66 & 48 & 36 & 36 & 81 & 45 & 0 & 44 \\ 71 & 44 & 32 & 41 & 44 & 17 & 44 & 0 \end{bmatrix}$	$\left(\frac{1}{432}\right)\begin{bmatrix} 566 \\ 487 \\ 381 \\ 386 \\ 478 \\ 317 \\ 356 \\ 293 \end{bmatrix}$	$\left(\frac{3264}{432}\right) = 7.5556$
6b	$\left(\frac{1}{432}\right)\begin{bmatrix} 0 & 63 & 29 & 53 & 45 & 33 & 17 & 33 \\ 63 & 0 & 66 & 102 & 99 & 78 & 71 & 87 \\ 29 & 66 & 0 & 48 & 81 & 36 & 20 & 44 \\ 53 & 102 & 48 & 0 & 72 & 72 & 44 & 68 \\ 45 & 99 & 81 & 72 & 0 & 69 & 53 & 60 \\ 33 & 78 & 36 & 72 & 69 & 0 & 41 & 57 \\ 17 & 71 & 20 & 44 & 53 & 41 & 0 & 32 \\ 33 & 87 & 44 & 68 & 60 & 57 & 32 & 0 \end{bmatrix}$	$\left(\frac{1}{432}\right)\begin{bmatrix} 273 \\ 566 \\ 324 \\ 459 \\ 479 \\ 386 \\ 278 \\ 381 \end{bmatrix}$	$\left(\frac{3146}{432}\right) = 7.2824$



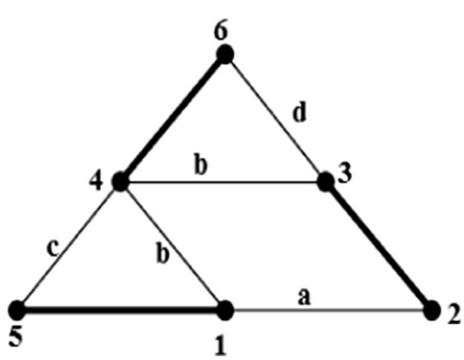
(a) 6-links Two-degree of freedom EGT-1



(b) 6-links Two-degree of freedom EGT-2



(c) Conventional graph of 6-links Two-degree of freedom EGT-1



(d) Conventional graph of 6-links Two-degree of freedom EGT-2

◀**Fig. 7** **a** Six-link two-degree-of-freedom EGT-1. **b** Six-link two-degree-of-freedom EGT-2. **c** Conventional graph of six-link two-degree-of-freedom EGT-1. **d** Conventional graph of six-link two-degree-of-freedom EGT-2

'Pair value (PV)' of the epicyclic gear trains and summation of all the pair values of EGTs results 'train value (TV)'. For example, EGTs with 4 links, 4 joints (two lower pairs AND two higher pairs), and single degree of freedom are considered for the detection of isomorphism in EGTs.

From Fig. 2a, kinematic pairs, i.e., 1–2, 1–3, 1–4, 2–3, and 2–4, where 1–2, 1–3, and 2–4 are turning pairs (or lower pairs), whereas 1–4 and 2–3 are gear pairs (or higher pairs). The degrees of kinematic pairs are 2, 2, and 3 for the turning pairs 1–2 and 1–3 and the gear pair 1–4, respectively, at the link 1. So, the joint value will be [1/(2 × 2×3)] = (1/12) = (6/72). Similarly, the joint values at links no. 2, 3, and 4 are (6/72), (12/72) and (12/72), respectively, as given in calculation chart in Fig. 3. Now, we computed the least path value (LPV) between two kinematic links as considering the links 1 and 2 with different paths in Table 1.

The least path value for kinematic links 1–2 for Path no 1 is 12/72. Similarly, least path value for links 1–3 and links 1–4 will be (18/72) and (18/72), respectively. These values will be represented as a_{12}, a_{13} and a_{14} of the matrix. Finally, from the proposed algorithm, the modified path matrix [MPM] will be calculated as shown in Table 2.

The pair values of link 1, 2, 3, and 4 are (48/72), (48/72), (66/72), and (66/72), respectively. Now, train value in Fig. 2a is the summation of all pair values of EGTs, i.e., (228/72). Applying same methodology as used in Fig. 2a for calculation of modified path matrix, pair value and train value, finally train values of corresponding Fig. 2b–f are (214/72), (228/72), (214/72), (248/72), and (248/72).

It was observed that the results shown in Table 2 and Fig. 2a, c were isomorphic. Similarly, Fig. 2b, d, e, f is isomorphic to each other (Table 2).

Example 2 Consider two five-link one-DOF epicyclic gear trains shown in Fig. 4a, b.

It was observed from the results shown in Table 3, the train values of two EGTs were different. So, it is clearly shown that Fig. 4c, d is non-isomorphic.

Example 3 Consider two conventional graphs of six-link one-DOF epicyclic gear trains in Fig. 5a, b

It was observed from the results shown in Table 4 that the train values of two EGTs were different. So, it clearly showed that Fig. 5a, b is non-isomorphic.

Table 6 Pair values and train values for different graph of six-link two-DOF epicyclic gear trains

Figure no.	Modified path matrix (MPM)	Pair value (PV)	Train value (TV)
7a	$\left(\frac{1}{144}\right)\begin{bmatrix} 0 & 32 & 29 & 17 & 32 & 41 \\ 32 & 0 & 36 & 41 & 56 & 60 \\ 29 & 36 & 0 & 21 & 45 & 36 \\ 17 & 41 & 21 & 0 & 33 & 33 \\ 32 & 56 & 45 & 33 & 0 & 57 \\ 41 & 60 & 36 & 33 & 57 & 0 \end{bmatrix}$	$\left(\frac{1}{144}\right)\begin{bmatrix} 151 \\ 225 \\ 167 \\ 145 \\ 223' \\ 227 \end{bmatrix}$	$\left(\frac{1138}{144}\right) = 7.90278$
7b	$\left(\frac{1}{144}\right)\begin{bmatrix} 0 & 36 & 30 & 18 & 36 & 42 \\ 36 & 0 & 36 & 42 & 60 & 60 \\ 30 & 36 & 0 & 18 & 42 & 36 \\ 18 & 42 & 18 & 0 & 30 & 30 \\ 36 & 60 & 42 & 30 & 0 & 54 \\ 42 & 60 & 36 & 30 & 54 & 0 \end{bmatrix}$	$\left(\frac{1}{144}\right)\begin{bmatrix} 162 \\ 234 \\ 162 \\ 138 \\ 222 \\ 222 \end{bmatrix}$	$\left(\frac{1140}{144}\right) = 7.9167$

Example 4 Consider two structural graphs of eight-link one-DOF epicyclic gear trains shown in Fig. 6a, b

It was observed from the results shown in Table 5 that the train values of two EGTs were different. Therefore, they were non-isomorphic. It was also validated by P. Yang et al. [31] based on ant algorithm for detection of isomorphism in EGTs.

Example 5 Consider two six-link two-DOF epicyclic gear trains shown in Fig. 7a, b.

It was observed from the results shown in Table 6 that the train values of two EGTs were different. So, it clearly showed that Fig. 7a, b is non-isomorphic.

Results and Discussion

As per the above discussions, if the resultant Train values and string of Pair values were found identical in numeric quantity then the two EGTs will be isomorphic for topological synthesis of geared mechanism, and otherwise they were distinct. The proposed methodology was computed on the basis of various examples from four-link, five-link, six-link, and eight-link one-degree-of-freedom EGTs and six-link two-degree-of-freedom EGTs. All the above examples have been found satisfactory results with earlier literature [29, 45, 49, 53, 56]. This was a theoretical approach to test the isomorphism among epicyclic gear trains and its inversions.

Conclusions

In this study, a theoretical approach has been evolved to identification of isomorphism of mechanisms of epicyclic gear trains. Modified path matrix approach was reliable and efficient method due to complex calculations. The topological synthesis of EGTs with different links and DOF was evaluated by this approach to demonstrate isomorphism behaviors of EGTs. The future study of this work may be extended by taking lower pair as sliding pair and higher degree of freedom of EGTs.

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